CMPF-B: Unit 2

Instructional Goals

1. **Differentiate position, distance and displacement**
   - Develop calculations of distance and displacement
     - Represent displacement as delta-x
2. **Differentiate clock readings and time intervals**
   - Establish *delta-t* to represent the time interval
     - Connect Δt and `delta-t`
3. **Understand slope as a rate of change**
   - Use the rate of change to find physical meaning in a graph
4. **Velocity as the rate of change for position over time interval**
   - Calculate average velocity as $\frac{delta-x}{delta-t}$
5. **Using a motion map to pictorially represent the motion of an object**
   - Connect the motion map representation with the computational representation
6. **Representing the motion of an object with a one- or two-argument function**
   - Construct `next-x()` as a function of current position `x`
   - Construct `next-x()` as a function of current position `x` and average velocity `v`
   - Introduce *Boolean* as a Pyret data type (alongside *Number*, *String*, and *Image*)
7. **Using multiple representations to describe the motion of an object**
   - Find relations between state diagrams, motion maps, programming functions, and graphs

Student Learning Objectives

CV.1 - I can represent the motion of an object using a computational representation in which a differential is added for each interval of time.

CV.2 - I can design and modify functions in Pyret to represent constant velocity motion and use them to make predictions.

CV.3 - I can differentiate between position, distance, and displacement.

CV.4 - I can find the velocity of an object using the slope of the position vs. time graph. I can interpret the initial value of a position vs. time graph. I can draw an average velocity line on the graph and calculate the average velocity.

CV.5 - I can create a mathematical representation (function) relating position, average velocity and time, and use it to solve problems.

CV.6 - I can represent the motion of an object moving with a constant velocity using multiple representations (words, motion maps, position vs time graphs, and velocity vs. time graphs, computational function, etc.) and use them to solve problems.

CV.7 - I can relate displacement to the area between the horizontal axis and the line on a velocity vs. time graph.

Constant Velocity: An Overview

This unit introduces students to using laboratory tools to make quantitative measurements, specifically of position and time, and many new representations available to them when making quantitative observations (e.g., data tables, motion maps, computer simulations, position vs. time graphs, and velocity vs. time graphs).

The unit begins with a paradigm lab, collecting measurements of the changing position of an object with relatively constant motion at various clock readings. Using this collected data, students create a motion map and a computational simulation of the object, resulting in the development of the relationship between the position at one moment in time and the next. Critically, students first discover the pattern of the object's next position at some future time being the *current position plus the change in position during that time interval*, and later revised to the *current position plus the product of the velocity and the time interval*. This progression leads to the operational definition of velocity as the ratio of the change in position over the change in time.

The ***differential representation*** of motion which is introduced through coding (contrasted with the more common parametric representation of motion) is central to the understanding of motion built in this unit. This idea — that position changes iteratively by some small amount Δx over some small time interval Δt — is a natural way to think about change and forms the basis of our computational representation of motion. Investigating the relationship between Δx and Δt naturally leads to the definition of *average velocity* as

$$\bar{\vec{v}} = \frac{\Delta \vec{x}}{\Delta t}.$$

An emphasis is placed early in the unit on collected data (stored in tables), computer simulations, and motion maps. This intense focus on the development of motion maps is intentional and important because motion maps are a more concrete representation of motion than graphs of position and time. By linking tabulated measurements, state diagrams, motion maps, and computational functions together so tightly, students are more ready to make the leap to the higher-order graphical representations later in the unit, and each subsequent unit.

By the end of the Constant Velocity unit, students are able to accurately describe the motion of objects with constant velocity, represent it through multiple representations, and predict the motion of objects with constant motion beyond the time they have observed the motion. They will also be able to identify objects which do *not* behave like they have constant motion - priming the deeper dive into changing motion and uniform acceleration in the next unit.

I. An Operational Definition of Motion

The combined models of motion (e.g., constant velocity, uniform acceleration, projectile motion, rotation, etc.) fall under the umbrella of "kinematic" models - the models which produce a *description* of an object's motion over a time interval. This is contrasted with "dynamic" models - models which explain the *cause* of an object's motion over a time interval. (i.e., force models).

Motion is defined as *the change in position of an object (or system) between two states*.

II. Vocabulary of Motion

It is extremely important the vocabulary we use is as clear as possible to make sure that we are not using ambiguous terms that can lead to more student confusion, and even more so we need to ensure that our students mean the word they are using rather than playing darts with the vocabulary and just tossing various words out in the hopes of landing on the right one. It is important to ask your students from time to time to define the term they use, especially early in the unit as they start to use vocabulary to help strengthen the connection to the concepts we are studying, even if they use terms correctly. Students will use the correct term at times while still not understanding *why* it is the correct term in the proper context. It will take extra time early on but lead to a stronger understanding and pay off in taking less time reteaching this vocabulary later.

Clock Reading and Time Interval

Clock Reading - this is the numerical value represented by the clock, such as the position of the second hand on an analog clock, or the numerical output on a digital clock. Clock readings expressed in seconds can be abbreviated as t(s). Clock reading would be the value of the time for a given state of the system. This is a *scalar* quantity as it *does not* require a direction to be defined.

Time Interval - it requires two clock readings to define a *time interval*. It is the difference in time from the initial clock reading (t_i) to the final clock reading (t_f). Time interval can be used interchangeably with ‘delta-t.’ This is a *scalar* quantity as it *does not* require a direction to be defined.

$$\Delta t = t_f - t_i$$

Position, Distance, and Displacement

Origin - arbitrarily selected reference point from which all positions will be measured. This could be referenced as ‘the equator’ for latitude, ‘prime meridian’ for longitude, or ‘the ground’ for height measurements. Horizontal motion allows for a much more arbitrary definition and can allow for nearly any location to be the origin but once selected, it is locked in for that situation. The object does *not* need to have been present at a location for that location to be a useful origin.

Distance - requires two points to be referenced and measure or calculate the length of the path between the two points. Distance can be measured as the path an object takes from one clock reading to another, “counting the steps” colloquially. This is a *scalar* quantity as it *does not* require a direction to be defined.

Position - location of an object as measured from the defined origin, including both the ‘distance’ away and the direction. For example, ‘3 meters, North’ is different from ‘3 meters, South' or '3 meters, East.' This is a position value for a specific state, just as clock reading is a time value for a specific state. These would be 'state values.' This is a *vector* quantity as it *does* require a direction to be defined.

Displacement – as with time interval, displacement requires two clock readings to be defined. Displacement is the difference between the position at an 'initial' clock reading and the position at a 'final' clock reading. Displacement is abbreviated as 'Δx.' This is a *vector* quantity as it *does* require a direction to be defined.

$$\Delta\vec{x} = \overrightarrow{x_f} - \overrightarrow{x_\iota}$$

Average Speed and Average Velocity

These are both ratios, and as such will be a much more challenging piece to define without a very strong understanding of the previously defined terms.

Average Speed - ratio of the distance traveled between two clock readings and the time interval between those same clock readings. This is a *scalar* quantity as it does not require a direction to be defined.

$$avg.speed = \frac{distance\ traveled}{time\ interval}$$

Average Velocity - ratio of the displacement traveled between two clock readings and the time interval between those same clock readings. This is a *vector* quantity as it *does* require a direction to be defined.

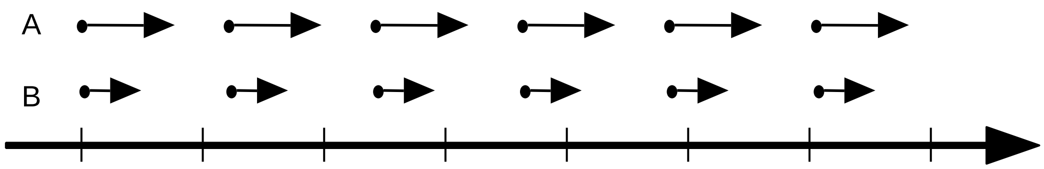
$$avg.velocity = \frac{displacement}{time\ interval}$$

Or using a more algebraic representation:

$$\bar{\vec{v}} = \frac{\Delta\vec{x}}{\Delta t}$$

III. Representational Tools

Motion maps form the basis for the computational representation of motion in this unit. Emphasize that the difference between dots is the displacement of the object for that particular interval of time. As the time interval decreases the displacement must also decrease if the object is to remain moving with the same rate of change.

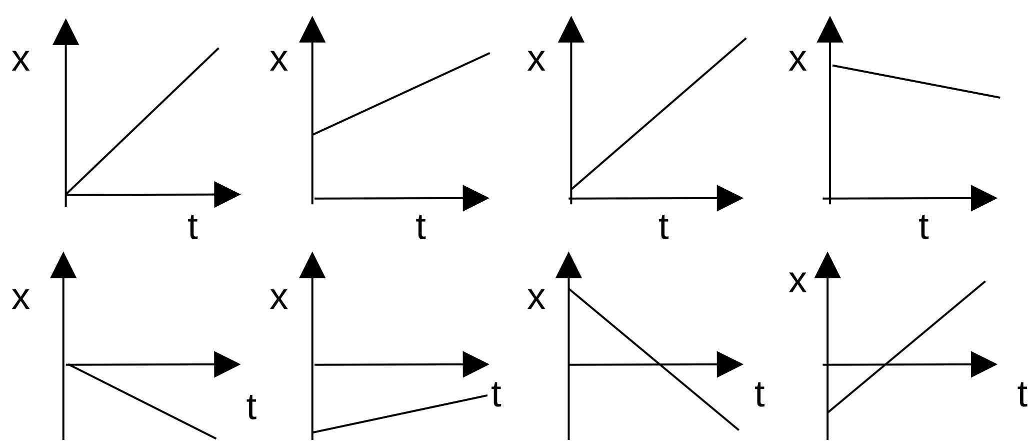


Students have been known to draw the motion map above and state that A was moving faster than B because the velocity vectors were longer. If object A is moving faster than object B, then the space between the dots should also be greater because A's change in position will be larger during each time interval.

Work on making motion maps to represent the position-time behavior of moving objects. Make sure that these semi-quantitative devices are faithful representations depicting where the object is at evenly-spaced clock readings.

1. The definition for *average velocity*, $\bar{\vec{v}} = \frac{\Delta \vec{x}}{\Delta t}$, should be used explicitly in discussing the physical significance of the slope of a position-time graph.

2. When discussing the meaning of the graphs, be sure to use a wide variety of examples.

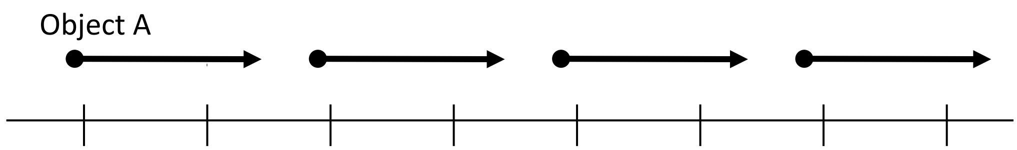


Induce the students to describe the motion in full detail (e.g., in the fourth graph, the object starts somewhere to the right of the zero position and moves to the left at constant speed).

3. Students will have to be taught how to manually produce a graph and do a mathematical analysis of the graph. Students have been conditioned to think of slope only as "rise over run" or Δy over Δx. They need to understand that the slope of any graph describes the rate of change in the physical quantity represented on the vertical axis with respect to the one represented on the horizontal axis.

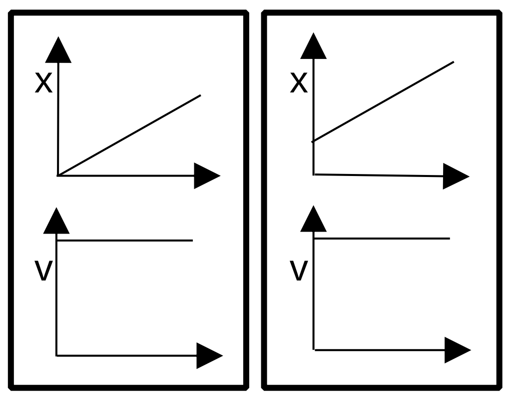
4. Make sure that they have a thorough grasp of the relationship between slope and velocity. The answer "1's slope is greater than 2's" is not a guarantee of understanding. It would be profitable to have students model the behavior of the object represented by a variety of graphs. If you have an ultrasonic motion detector, this is great fun!

5. Make sure that students can, given a verbal description, an algebraic statement, a function in Pyret, an $\vec{x}$ vs t graph, or a motion map, recreate the other four representations.



Given the motion map above, they should be able to write a verbal description of the motion, express the relationship $\vec{x} = \vec{v}t + \overrightarrow{x_o}$ and draw the graph at right.

6. Be sure to make the connection between $\vec{x}$ vs t graphs and $\vec{v}$ vs t graphs. "Stacking" the curves by placing the $\vec{v}$ vs t graph directly underneath the $\vec{x}$ vs t graph helps to illustrate this relationship.



Make the point that the $\vec{v}$ vs t graph yields no information about starting point. In the two stacks shown, two different $\vec{x}$ vs t graphs can be represented by the same $\vec{v}$ vs t graph.

7. Make the point that the area under a $\vec{v}$ vs t graph represents the displacement, the $\Delta\vec{x}$ of the object. This could be both (+) and (-). Avoid always using the trivial case.

8. Key understandings about computation in physics raised in this unit are:
   A. Computers allow us to do many repeated calculations which would be too time-consuming to do by hand.
   B. Computers can simulate (represent) physical behavior and use those simulations to make predictions about physical phenomena.
   C. Instructions which should be repeated multiple times can be written using functions

Sequence

1. Lab 1 — Buggy Lab
2. Activity 1 — Simulated Motion
3. Activity 2 — Advanced Simulated Motion
4. Worksheet 1 — Motion Maps
5. Worksheet 2 — Position-Time Graphs
6. Worksheet 3 — Two Bicycles
7. Lab 2 — Graph Matching with Motion Detectors
8. Activity 3 — Multiple Objects
9. Lab 3 — Colliding Buggies
10. Worksheet 4 — Distance vs. Displacement
11. Reading 1 — Vocabulary of Motion
12. Worksheet 5 — Velocity-Time Graphs
13. Activity 4 — Multiple Representations of Motion
14. Activity 5 — Rocket Lander Game
15. The Model So Far

Lab 1: Buggy Lab

Part I

Observe the motion of the buggy and answer the following questions:

1. Does the buggy move? How do you know?

2. Draw a series of state diagrams for the buggy. What quantity or quantities change from diagram to diagram?

Part II

Each time the metronome ticks, mark the position of the buggy on the receipt tape. Do this for a total of ten ticks. Create a table of the results in the space below.

3. Using the above results, predict the position of the buggy on the eleventh, twelfth and thirteenth ticks of the metronome. Perform an experiment to test these predictions.

Tick	Predicted Position	Measured Position
11		
12		
13		

4. Does the data follow a pattern? If so, how is this pattern represented in the state diagrams?

Unit 2 Activity 1: Simulated Motion

Part I

The next representation of the motion of the buggy will be a computer simulation based on the collected data. To do this, we must first represent the motion of the buggy with a ***function***.

The ***purpose statement*** of this function will read:

```
# Consumes the current position of the buggy and
# produces its position at the next metronome tick.
```

1. What does this function need to take as an input? What does this function produce as an output?

2. The name of this function will be `next-x` since it produces the position of the buggy at the next metronome tick. Write a ***contract*** for `next-x`.

   `next-x :: ________________ -> ________________`
   Input "Type" Output "Type"

3. In the left column of the table draw a state diagram for the buggy with its position labeled. In the middle column write the corresponding Pyret example that consumes the position of the buggy at that state. In the right column draw a state diagram for the next position of the buggy, again with its position labeled.

Current State Diagram	Pyret Examples	Next State Diagram
	`examples:`	
	`end`	

4. Given the current position of the buggy, how can the next position be calculated? Write this as an algebraic expression.

5. Obtain a ***Function Design*** and fill it out for this `next-x` function.

Part II

Load the program https://tinyurl.com/U2-Simulated-Motion

The notation ... indicates that text must be added before the program will run. In addition to the information written in the function design, there are two initial parameters that must be given values: `delta-t` and `initial-position`.

6. Read the comments for `delta-t` and `initial-position` and enter the values used during the buggy lab.

7. Use the function design from Part I to complete the `next-x` function in the code.

8. Run the simulation and describe what happens. Is this how you expected your simulation to work? Explain.

Unit 2 Activity 2: Advanced Simulated Motion

Load your saved buggy simulation from Activity 1.

1. Your simulation output from Activity 1 includes a table of values with the headings tick, time, and position. Copy this table below. What do each of these headings indicate?

   Then change your delta-t to 1 second and run your simulation again. Record the new data in the table below.

Data table for delta-t = 2 sec				Data table for delta-t = 1 sec		
tick	time	position		tick	time	position

2. What does tick "2" tell you regarding the buggy? Is the information the same for both tables? Explain why or why not.

3. What does time "4 seconds" tell you regarding the buggy? Is the position the same? Is the tick the same? Explain.

4. As we keep changing the delta-t value, the position information is no longer accurate! Where should your buggy be with the given delta-t? **The third column will be used in question 5.** (Add more delta-t examples until you see a pattern!)

delta-t (s)	expected change in position (cm)	
2		
1		
0.5		
10		

5. Think back to your whiteboard discussion. We saw that the delta-t we used had an effect on how far our buggy traveled in that time interval. What is the pattern that you noticed in the table? What common calculation could you do that uses the delta-t value and gives us the expected position?

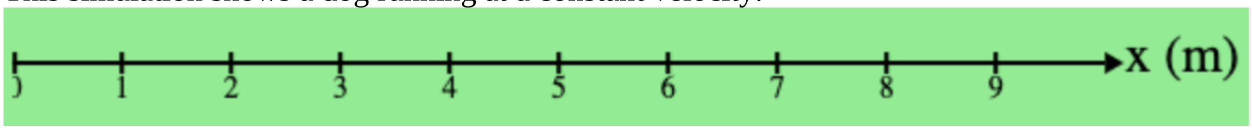
6. We need to change our next-x function so that it returns the correct expected position for each clock reading regardless of the delta-t value. **Our function purpose statement and contract will stay the same as before,** but we need new examples and a new function definition. Complete the new Function Design on paper before you make any changes to your code.

7. At the top of your function design there is a request for defined identifiers. What constant values will you need for your calculation?

8. When you can change your delta-t to any value and the position of the car at a particular clock reading is the same you will know you have successfully modeled the motion of your buggy into a computer simulation. Congratulations!

Unit 2 Worksheet 1: Motion Maps

Use this url: The student code can be found at https://tinyurl.com/U2-Motion-Map
This simulation shows a dog running at a constant velocity.



1. Complete the `next-x` function as in previous simulations. Set an initial position and velocity that will make the dog appear on the background above, each mark representing one meter. Sketch the image output shown in the interactions window after the reactor window closes.

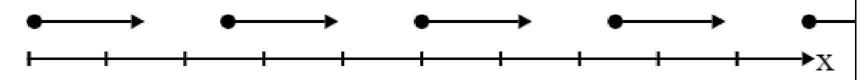
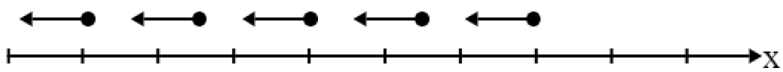
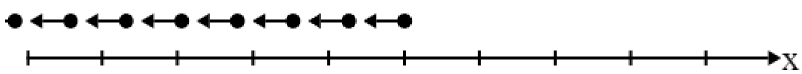
2. Change the velocity and initial position, run the simulation, and sketch the new image output.

3. The images are ***motion maps***. What information about the motion of the dog can be determined from looking at the motion map? How is this information represented?

4. Predict what the motion map will look like for the following initial parameters. Test these predictions with the simulation and draw the outcome in the table below.
   a. $x_o = 1$, $v = 2$ m/s
   b. $x_o = 4.5$, $v = -1.5$ m/s
   c. $x_o = 5$, $v = 1.25$ m/s

Prediction	Outcome
a. x	x
b. x	x
c. x	x

5. Determine the initial position and velocity of the dog in each case from the motion maps below. Use the simulation to check your work.

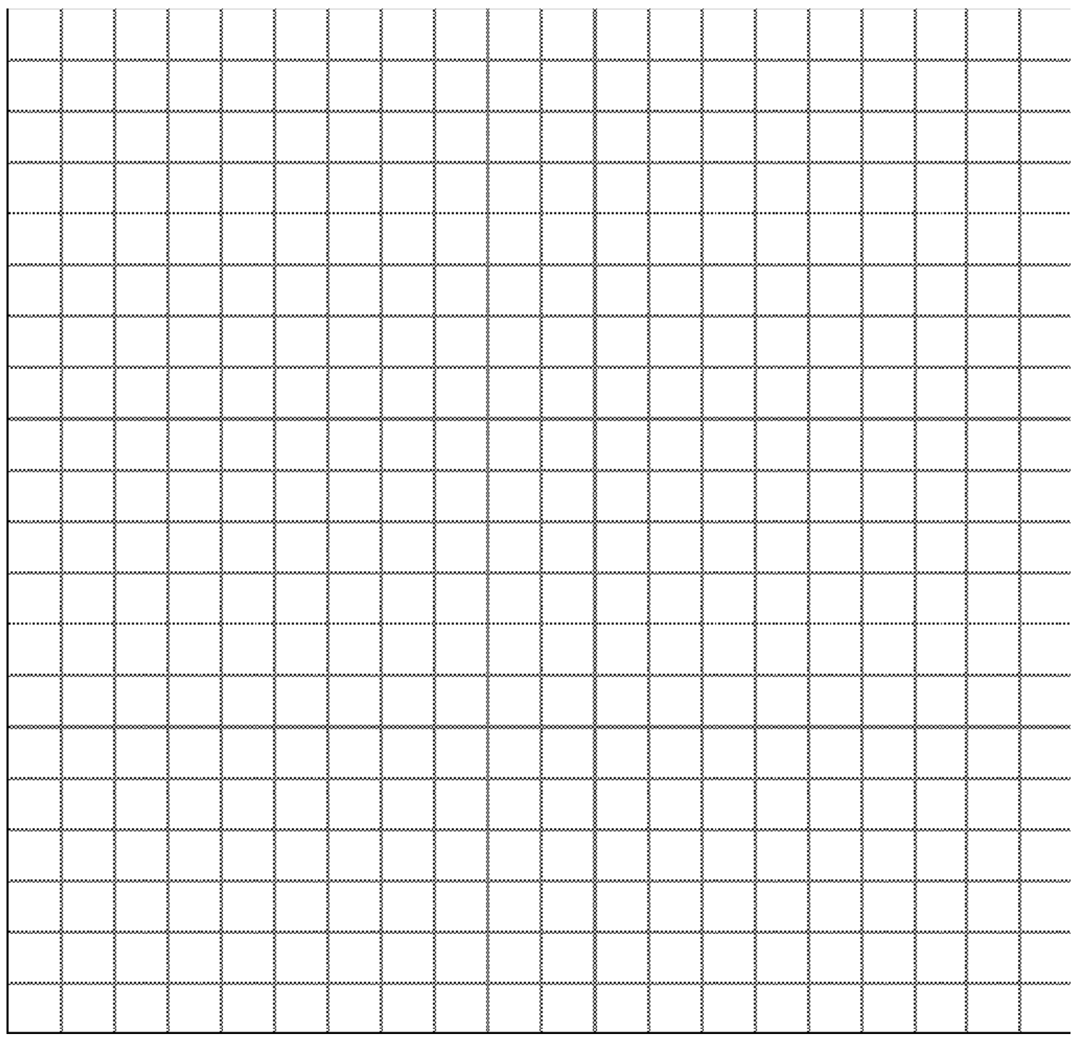
Motion Map	Initial Conditions
	x_o = ____________, v = ____________
	x_o = ____________, v = ____________
	x_o = ____________, v = ____________

6. What would change about these motion maps if the position was marked every two seconds, instead of every one second? How do you know?

7. What would change about these motion maps if the position was marked every 0.5 seconds? How do you know?

Unit 2 Worksheet 2: Position-Time Graphs

1. Plot points of position and time for two buggies on the graph below (you may need to share information from another lab group). Draw a line of best fit for each data set.

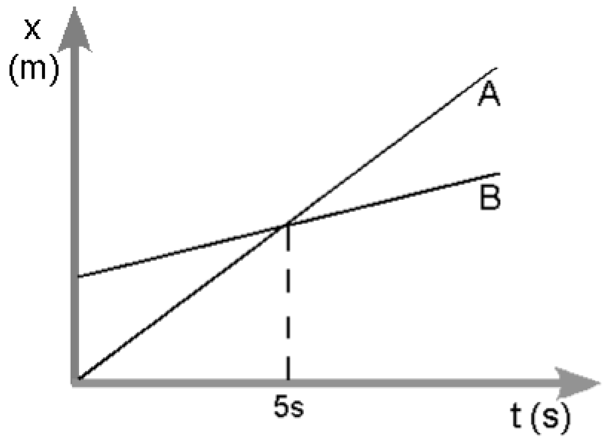


2. What might the vertical intercept of each line represent about the motion of the buggy?

3. What is the slope of each line? What would be the units for this slope? What could it represent for the motion of the buggies?

Unit 2 Worksheet 3: Two Bicycles

1. Consider the position vs. time graph below for cyclists A and B.



a) Do the cyclists start at the same point? How do you know? If not, which is ahead?

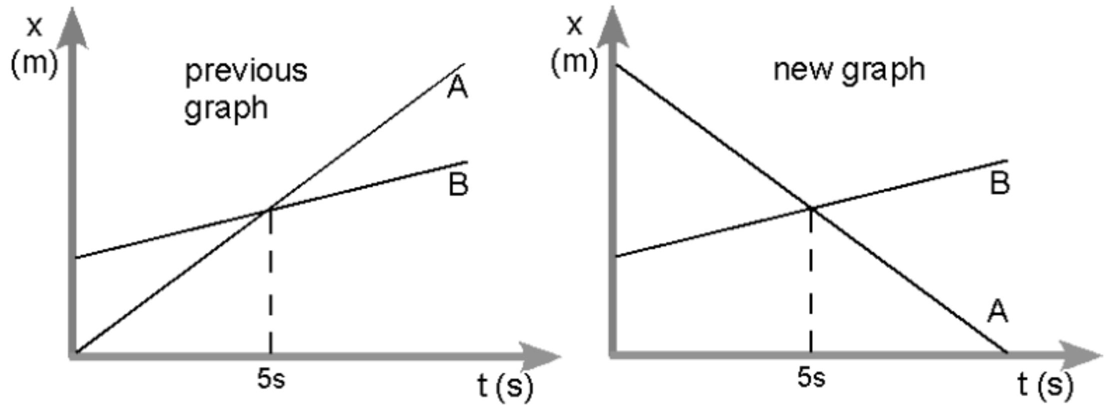
b) At t= 7s, which cyclist is ahead? How do you know?

c) Which cyclist is travelling faster at t = 3s? How do you know?

d) Are their velocities equal at any time? How do you know?

e) What is happening at the intersection of lines A and B?

2. Consider the new position vs. time graph below for cyclists A and B.



a) How does the motion of the cyclist A in the new graph compare to that of A in the previous graph from page one?

b) How does the motion of cyclist B in the new graph compare to that of B in the previous graph?

c) Which cyclist has the greater speed? How do you know?

d) Describe what is happening at the intersection of lines A and B.

e) Which cyclist traveled a greater distance during the first 5 seconds? How do you know?